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DESIGN OF A SOLAR EPHEMERIS SYNTHESIZER FOR LES-8/9

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The various design considerations of a solar ephemeris synthesizer for the LES-8/9 stationkeeping system are discussed here. Different approximations are used to reduce the amount of hardware required for building the synthesizer. The overall accuracy of the synthesizer is about 0.01 deg for orbits inclined up to 5.0 deg from the ecliptic plane. For orbits inclined at greater angles up to 10 deg, the short term accuracy is 0.01 deg/day and the long term accuracy is about 0.05 deg/year. A few ephemerides generated by the synthesizer are graphically illustrated.

Accepted for the Air Force
Joseph J. Whelan, USAF
Acting Chief, Lincoln Laboratory Liaison Office

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DESIGN OF A SOLAR EPHEMERIS SYNTHESIZER FOR LES-8/9

I. INTRODUCTION

An autonomous stationkeeping system for a synchronous satellite is required to control the mean longitude of the satellite. The daily variation of the longitude about a mean, which results from the eccentricity and inclination of the orbit to the equator, is uncontrolled and goes through a well predictable cycle. The input required for the above stationkeeping system is the angular position of the satellite in its orbit plane. When the eccentricity of the orbit is not required to be controlled, only two measurements at opposite points in the orbit are sufficient. For a synchronous satellite in circular orbit, the angular position can be determined by observing the azimuthal component of the sun's motion from a perfectly oriented satellite, that is, a satellite having no attitude errors. The sun's motion need not be observed continuously, but it is sufficient to know the time of sun transit across two azimuth transit sensors looking in opposite directions. Time is kept accurately by a stabilized frequency source having a short term stability of 0.5 sec/day and a long term stability of 2.5 sec/year. Thus, the orbit angle of the satellite can be computed at the time of sun transit by comparing the observed transit time with the expected transit time. The expected transit time is a function of inclination and ascending node of the orbit. The mean interval between the transit times of the sun across a transit sensor over one year is 24 hours. The difference between the mean transit time and expected transit time is a function having a period of one year. This function can alternatively be expressed as an angle rather than time with the usual conversion factor of 360°/24 hr. The solar ephemerides for observation from a perfectly oriented satellite are usually expressed as angular deviations at regular time intervals, whereas the above function gives the time difference between the expected and mean transit times at the expected transit times. In other words, the ephemerides are samples of

angular deviations at uniform time intervals, whereas the function considered here is a sample of the time delay at non-uniform time intervals. The function when expressed in angular units will be indistinguishable from the solar ephemeris within our desired accuracy. Therefore, without making any further distinction we shall refer to the time function expressed in angular units as the solar ephemeris and a synthesizer for the function as an ephemeris synthesizer.

In the above discussion it was assumed that the satellite is perfectly oriented. This is true only within a fraction of a degree for a three axis attitude controlled satellite similar to LES-8/9. Therefore, the satellite serves as a non-stationary platform. However, the correction for sun observation can be conveniently applied for small deviations of the satellite attitude [1]. The error due to eccentricity can be averaged out to a high degree by taking the average of an even number of consecutive observations taken at opposite ends of the orbit.

For the LES-8/9 satellites, which will be nominally in the ecliptic plane, it is sufficient to be able to generate ephemeris for orbits inclined up to ten degrees from the ecliptic plane. The short term accuracy of the synthesizer is desired to be good enough to give a high fuel efficiency for the stationkeeping system. For a $\pm$ 0.1 deg stationkeeping system a desirable value of the short-term accuracy is about 0.01 deg [2]. The long term error, which is not very much affected by the averaging scheme and has a very small effect on the fuel efficiency, causes a slow drift of the satellite. For orbits having inclination up to 5.0 deg from the ecliptic, the desired long term accuracy is 0.01 deg. For orbits at greater inclinations the long term error could be allowed to be larger at the cost of a slow station drift. The limiting case of 10 deg inclination the drift is about 0.05 deg/year. This tolerance allows a simple design and a small overall size for the synthesizer.

In this note the various design considerations for the synthesizer are discussed. Solar ephemerides synthesized for a few different orbits by the synthesizer are illustrated in the appendix.

II. NOTATIONS

The following notations are used in this note.

r = radius vector from the satellite to the sun

μ = GM_e, where G is the gravitational constant and M_e is the mass of the earth

a = semi-major axis of the orbit

e = eccentricity of the orbit

f = true anomaly of the orbit

n = mean angular velocity

ℓ = mean anomaly

ω = argument of the perigee

J_n = Bessel function of order n

u = $\omega + f$

$a_1, a_2 \ldots$ = eccentricity coefficients in the expansion of the true anomaly as a function of mean anomaly

t = time in days

$\psi_1(\ell)$ = the difference between true anomaly and mean anomaly for the ecliptic plane

$[\ \ ]^T$ = transpose of a vector

x, y, z = rectangular coordinates of a vector

Ω = right ascension of ascending node

i = inclination of the orbit

θ = error in position measurement after averaging measurements at two opposite ends of the orbit

$I = \frac{i^2}{4} + \frac{i^4}{24}$ = inclination coefficient

$\psi_2(u)$ = part of a solar ephemeris resulting from the inclination of the satellite orbit

$\psi_3(u)$ = an error in $\psi_2(u)$ approximation

$\psi_4(u)$ = an error due to solar parallax from synchronous orbit

$\psi_5(k)$ = an error due to jump summation.

Int(s) = integral part of "s"

α = angle of the sun-earth vector

β = earth satellite angle

$\tau = 2\pi \frac{366.27}{365.27} t$, a time variable

γ = earth-sun angle subtended at the satellite

η = inclination of the equatorial plane with the ecliptic = 23.44°

e_s = eccentricity of satellite orbit

δ_{ij} = Kroneker delta, 1 for $i = j$, 0 for $i \neq j$

$\bar{\delta}_{ij} = 1 - \delta_{ij}$

III. SUN AZIMUTH FOR ECLIPTIC ORBIT

For our desired accuracy, the motion of a satellite around the sun may be approximated by the motion of the earth around the sun. The small inaccuracy in this approximation may be treated as a parallax error. It is assumed that the satellite is in a circular orbit and perfectly oriented, so that its equatorial plane is coplanar with the orbit plane.

The motion of the earth around the sun may be treated as a two body classical problem. Therefore,

$$\ddot{\hat{r}} = -\mu \frac{\hat{r}}{r^3} \quad , \qquad (3\text{-}1)$$

where $\hat{r}$ is the sun-earth vector and μ is the product of the gravitational constant and the mass of the sun.

The solution of the above equation is the ellipse [3]*

$$r = \frac{a(1-e^2)}{1+e \cos f} \quad , \qquad (3\text{-}2)$$

where a is the semi-major axis, e is eccentricity and f is true anomaly of the earth orbit.

The mean angular velocity n is given by

$$n = \mu^{1/2} a^{-3/2} \quad . \qquad (3\text{-}3)$$

The mean anomaly ℓ is given by

$$\ell = nt \quad , \qquad (3\text{-}4)$$

where t is the time measured from the initial perigee passage. A series solution of Eq. (3-1) is given by [3].

$$f = \ell + 2\sum_{s=1}^{\infty} \frac{1}{s} \left\{ J_s(se) + \sum_{p=1}^{\infty} \beta^b \left[J_{s-p}(se) + J_{s+p}(se) \right] \right\} \sin s\ell \quad , \qquad (3\text{-}5)$$

---

* The number within the square bracket [] refers to the reference number.

where J_s is Bessel's function of order s, and

$$\beta = \frac{1}{e}\left(1 - \sqrt{1-e^2}\right) \quad . \tag{3-6}$$

Equation (3-5) written more explicitly gives [3]

$$\begin{aligned} f = \; & \ell + \left(2e - \frac{1}{4}e^3 + \text{[illegible]} + \ldots\right) \sin \ell \\ & + \left(\frac{5}{4}e^2 - \frac{11}{24}e^4 + \ldots \;\; \sin\right) 2\ell \\ & + \left(\frac{13}{12}e^3 - \frac{43}{64}e^5 + \ldots\right) \sin 3\ell \\ & + \left(\frac{103}{96}e^4 - \ldots\right) \sin 4\ell + \left(\frac{1097}{960}e^5 - \ldots\right) \sin 5\ell \\ & + \ldots . \\ = \; & \ell + a_1 \sin \ell + a_2 \sin 2\ell + a_3 \sin 3\ell + a_4 \sin 4\ell + \ldots \quad , \end{aligned} \tag{3-7}$$

where $a_1, a_2, a_3, \ldots$ are the coefficients of the eccentricity terms and $\ell = \frac{2\pi t}{365.27}$. Therefore the variation of the sun azimuth angle from the mean sun angle is given by

$$\begin{aligned} \psi_1(\ell) &= f - \ell \\ &= a_1 \sin \ell + a_2 \sin 2\ell + a_3 \sin 3\ell + a_4 \sin 4\ell + \ldots \end{aligned} \tag{3-8}$$

Notice that $\psi_1(\ell)$ is an odd function around $\ell = n\pi$, $n = o, \pm 1, \pm 2, \ldots$, but it is not an even function around $\ell = (n + \frac{1}{2})\pi$, $n = o, \pm 1, \pm 2, \ldots$; therefore, the sign of the function cannot be changed by changing the phase by a an angle π.

IV. SUN AZIMUTH FOR NON-ECLIPTIC ORBITS

The satellite's plane of motion is assumed to be the XY plane in Fig. 1. OS is the satellite-sun vector. The sun azimuth is obtained by projecting the vector OS on the XY plane. The coordinates of the sun in XYZ frames are obtained by the following transformation

(1) Rotating $[r,0,0]^T$ by $-u$ around Z"

(2) Rotating $[r',0,0]^T$ by $-i$ around X'

(3) Rotating $[r'',0,0]^T$ by $-\Omega$ around Z

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos u & \sin u & 0 \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} r\cos u\cos\Omega - r\sin u\sin\Omega\cos i \\ r\cos u\sin\Omega + r\sin u\cos\Omega\cos i \\ r\sin u\sin i \end{bmatrix} \tag{4-1}$$

For convenience let the X axis be coincident with the ascending node, i.e., $\Omega = 0$, Eq. (4-1) becomes:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} r\cos u \\ r\sin u\cos i \\ r\sin u\sin i \end{bmatrix} \tag{4-2}$$

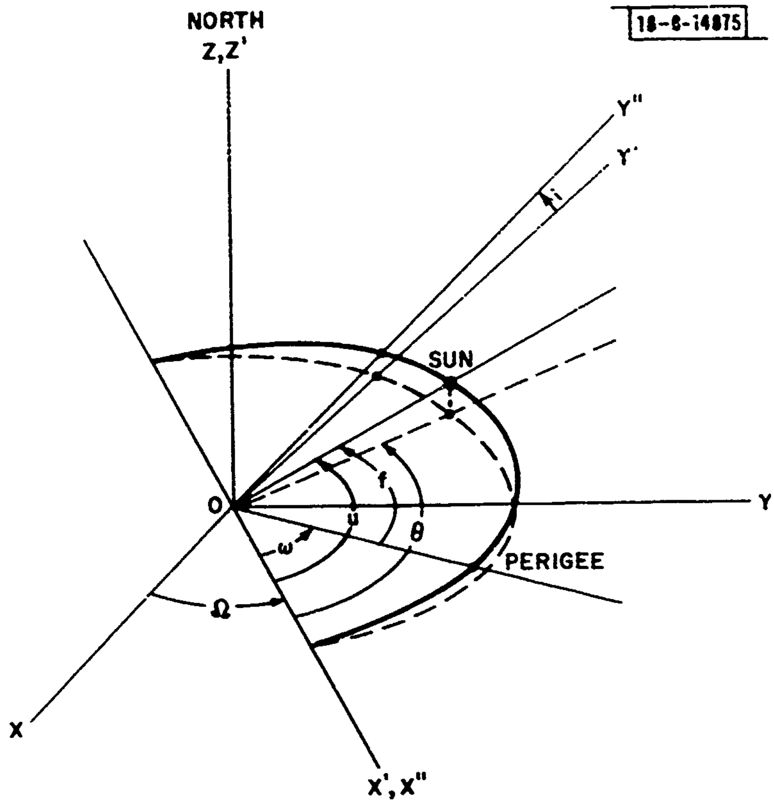

Fig. 1. Geometry of sun's motion observed from a synchronous circular satellite.

From Eq. (4-2), the sun azimuth is given by

$$\theta = \sin^{-1} \frac{y}{\sqrt{x^2+y^2}}$$

$$= \sin^{-1} \frac{\sin u \cos i}{p(i)} \quad , \qquad (4\text{-}3)$$

where

$$p(i) = \left(1 - \sin^2 u \sin^2 i\right)^{1/2} \quad .$$

The difference between the sun azimuth angles of an ecliptic orbit and a non-ecliptic orbit is given by

$$\psi_2(u) = u - \theta$$

$$= \sin^{-1} \left\{\frac{1}{2}(1 - \cos i) \frac{\sin 2u}{p(i)}\right\} \qquad (4\text{-}4)$$

$$= \frac{1}{2}(1 - \cos i) \frac{\sin 2u}{p(i)} + \frac{1}{48}(1 - \cos i)^3 \frac{\sin^3 2u}{p(i)^3}$$

$$= \left(\frac{1}{4} i^2 + \frac{1}{24} i^4\right) \sin 2u - \frac{1}{32} i^4 \sin 4u + \ldots \quad . \qquad (4\text{-}5)$$

From Eq. (4-4), $\psi_2(u)$ can be shown to be extremum at

$$u = \pm \tan^{-1} \left\{\frac{\cos i - 1}{(\cos i)^{1/2}}\right\} \quad .$$

V. APPROXIMATION OF THE SUN AZIMUTH

The inclination correction term for the orbit given by Eq. (4-5) is not very convenient for building a simple solar ephemeris synthesizer. Let $\psi_2(u)$ be approximated by the first term of Eq. (4-5). Therefore, the approximate $\psi_2(u)$ is given by

$$\overline{\psi_2}(u) = \left(\frac{i^2}{4} + \frac{i^4}{24}\right) \sin 2u$$

$$= I(i) \sin 2u \quad , \tag{5-1}$$

where

$$I(i) = \left(\frac{i^2}{4} + \frac{i^4}{24}\right) \quad .$$

The values of $I(i)$ are tabulated for various inclinations in Table I.

The maximum error in $\overline{\psi_2}(u)$ approximation is about $\frac{i^4}{32}$, and it is given in Table II for various inclinations.

From Eq. (3-7)

$$I(i) \sin \ell = \frac{I(i)}{a_1}\left[\psi_1(\ell) - a_2 \sin 2\ell - a_3 \sin 3\ell - a_4 \sin 4\ell \; \ldots\right] \quad . \tag{5-2}$$

Let $\ell = 2u$, then

$$\overline{\psi_2}(u) = \frac{I(i)}{a_1}\left[\psi_1(2u) - a_2 \sin 4u - a_3 \sin 6u - a_4 \sin 8u + \ldots\right]$$

$$= \frac{I(i)}{a_1}\psi_1(2u) - \frac{I(i)}{a_1}\left[a_2 \sin 4u + a_3 \sin 6u + a_4 \sin 8u + \ldots\right] \tag{5-3}$$

TABLE I

TABLE OF THE INCLINATION FACTOR $I(i) = \frac{i^2}{4} + \frac{i^4}{24}$

Inclination i deg	I(i) deg
0	0
2.5	0.0271
5	0.109
10	0.435
20	1.739
23.5	2.33

TABLE II

TABLE OF THE ERROR IN $\overline{\psi_2}(u)$ APPROXIMATION DUE TO NEGLECTING FOURTH AND HIGHER HARMONICS

Inclination "i" deg	Error $\approx \frac{i^4}{32}$ deg
0	0
2.5	0.0004
5	0.0016
10	0.0066
20	0.0264
23.5	0.0501

Let $\overline{\psi_2}(u)$ be approximated by the first term in Eq. (4-8). The maximum error $\psi_3(u)$ in this approximation satisfies

$$\psi_3(u) \leq \frac{I(i)}{a_1}\left[|a_2| + |a_3| + |a_4| + \ldots\right] \quad . \tag{5-4}$$

For the earth orbit $e = 0.01675$, and the eccentricity terms in Eq. (3-7) are

$$\begin{aligned} a_1 &= 3.350 \times 10^{-2} \quad , \\ a_2 &= 3.506 \times 10^{-4} \quad , \\ a_3 &= 5.09 \times 10^{-6} \quad , \ldots \quad . \end{aligned} \tag{5-5}$$

Using the above in Eq. (5-4), we get

$$\begin{aligned} \psi_3(u) &\leq I(i) \times 1.05 \times 10^{-2} \text{ rad} \\ &\leq 0.603\, I(i) \text{ deg} \quad . \end{aligned} \tag{5-6}$$

Table III gives the maximum value of $\psi_3(u)$ for various inclinations. The overall variation of the sun azimuth due to the earth's eccentric orbit and the inclination of the satellite's orbit plane with the equator is approximated by

$$\begin{aligned} \psi(t) &= \psi_1\left(\frac{2\pi t}{365.27}\right) \\ &\quad + \frac{I(i)}{a_1}\psi_1\left(\frac{4\pi t}{365.27} + 2\omega\right) \quad , \end{aligned} \tag{5-7}$$

where ω is the argument of the perigee (see Fig. 1).

In the above discussion the effect of parallax due to observations from the synchronous orbit has not been considered. The small parallax correction is necessary because in the Eq. (5-7) the synchronous distance is assumed to be negligible compared to the sun-earth distance. The maximum solar parallax observed from a synchronous orbit in the ecliptic plane is:

TABLE III

TABLE OF MAXIMUM ERROR $\psi_3(u)$ IN THE $\overline{\psi_2}(u)$ APPROXIMATION

i deg	$\psi_3(u)$ deg
0	0
2.5	0.0003
5	0.0011
10	0.0046
20	0.0182
23.5	0.0252

TABLE IV

MAXIMUM ERROR IN INCLINATION CORRECTION TERM
DUE TO JUMP SUMMATION

i deg	a_1/I	ψ_5 deg
0	∞	0
2.5	70.4	0.00002
5.0	17.6	0.003
6.6	10.0	0.010
10.0	4.4	0.050
14.8	2	0.240

$$\left[\psi_4(t)\right]_{max} = \frac{2R_{syn}}{A.U.} = \frac{2 \times 4.2165 \times 10^4}{1.496 \times 10^8}$$

$$= 0.03230 \quad \text{deg} \quad , \tag{5-8}$$

where R_{syn} is the synchronous radius and A.U. is the astronomical unit (mean earth-sun distance). The solar parallax is almost constant, if the measurement for the sun azimuth is performed for the same earth-sun angle subtended at the satellite. This constant error can be eliminated by either taking the average of measurements at two opposite points in the orbit or by adjusting the alignment of sun sensors.

Another source of error in using Eq. (5-7) for the synthesis of solar ephemeris is due to discrete storage of the function $\psi_1(t)$. Differentiating Eq. (3-8)

$$\frac{\delta\ \psi_1(\ell)}{\delta t} = \frac{2\pi}{365.27}\left[a_1 \cos \ell + 2a_2 \cos 2\ell + 3a_3 \cos 3\ell + \dots\right] \tag{5-9}$$

Using the constants $a_1, a_2, a_3, \dots$ from (5-5) in Eq. (5-9)

$$\left[\delta\ \psi_1(\ell)\right]_{max} = \frac{2\pi}{365.27}\left(a_1 + a_2 + a_3 + \dots\right)$$

$$= 0.0338\ \delta t \quad \text{deg} \quad . \tag{5-10}$$

If the function $\psi_1(\ell)$ be sampled at $\frac{1}{2}$ day intervals then the maximum difference between the sampling intervals is 0.017 deg, and the maximum error due to discrete storage is 0.0084 deg.

VI. ERRORS DUE TO ORBIT PERTURBATIONS

The orbit of a synchronous satellite changes significantly over its lifetime due to geopotential, luni-solar potential and solar radiation pressure [4,5,6]. The effect of other perturbations are small. The solar ephemeris for the satellite is affected by the changes in orbital elements. As an example consider a synchronous satellite in an eccentric orbit in the ecliptic plane (Fig. 2). From Eq. (3-7) the angle of the sun-earth vector OE is given by

$$\alpha = \ell - \ell_o + \sum_{k=1}^{\infty} a_k \sin k(\ell - \ell_o) \quad , \tag{6-1}$$

where ℓ_o is the mean anomaly for the direction OP and $\ell = \frac{2\pi t}{365.27}$. Similarly, the earth satellite angle SE is given by

$$\beta = \tau - \tau_o + \sum_{j=1}^{\infty} b_j \sin j(\tau - \tau_o) \quad , \tag{6-2}$$

where τ_o is the angle between the earth earth-satellite vector when the satellite is at its perigee and the initial sun-satellite vector OP. The variable $\tau = 2\pi \frac{366.27t}{365.27}$ and b_j's are the coefficients similar to a_k's when the eccentricity e is substituted by the eccentricity of the satellite orbit e_s.

From (6-1) and (6-2), the earth-sun angle at the satellite is given by

$$\begin{aligned}
\gamma &= \beta - \alpha \\
&= (\tau - \ell) + (\tau_o - \ell_o) + \sum_{k=1}^{\infty} \left\{ a_k \sin k(\ell - \ell_o) + b_k \sin k(\tau - \tau_o) \right\} \\
&= (\tau_o - \ell_o) + 2\pi \frac{t}{365.27} \\
&\quad + \sum_{k=1}^{\infty} \left\{ a_k \sin \frac{2\pi k}{365.27} (t - t_e) + b_k \sin 2\pi k \frac{366.27}{365.27} (t - t_s) \right\}
\end{aligned} \tag{6-3}$$

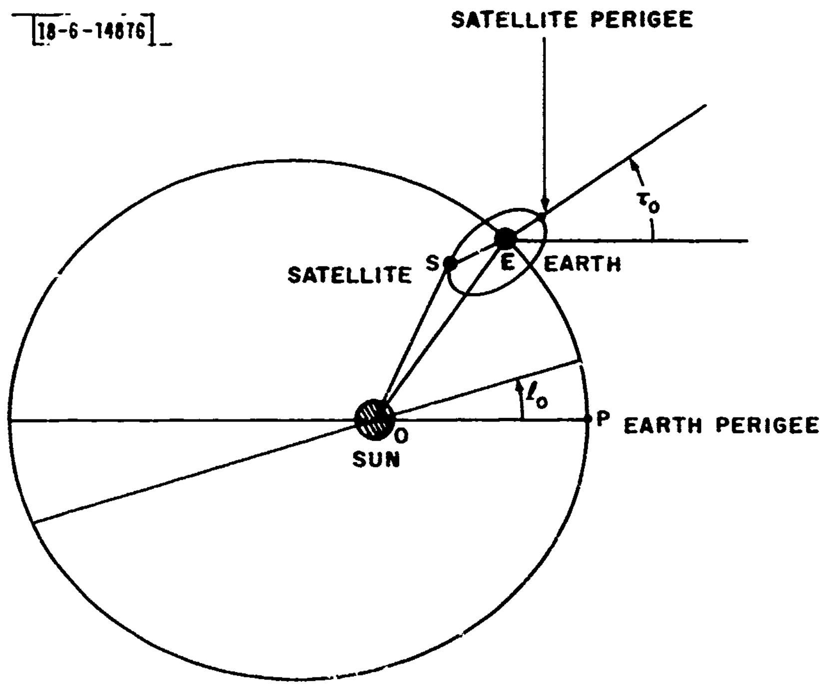

Fig. 2. Geometry of synchronous satellite orbit in ecliptic plane.

If it is assumed that the satellite has no pitch, roll or yaw error, then the successive times of the sun coincidence by a sun azimuth sensor on the satellite are obtained by solving (6-3) for $\gamma = \gamma_o + 2n\pi$, $n = 0, 1, 2, \ldots$, γ_o is the angle between the azimuth sensor field of view and the nominal pitch direction of the satellite. A general series solution or a closed form solution of Eq. (6-3) is a non-trivial problem. However, it is seen in (5-5) that a_k's converge to zero very rapidly. Similarly even for an eccentricity of 0.01 the b_k's will converge to zero quite fast. To eliminate the effect of the variation due to the eccentricity of the satellite's orbit, the sun azimuth angle is measured at opposite ends of the orbit, that is at $\gamma = \gamma_o + 2n\pi$ and $\gamma = \gamma_o + (2n + 1)\pi$, $n = 0, 1, 2, \ldots$ The maximum error in the position measurement of the satellite when the average of two such measurements are taken is given by

$$\theta_{max} \simeq b_i \left(\frac{2\pi}{365.27} + \frac{a_1}{365.27}\right)$$

$$\approx \frac{4\pi e_s}{365.27} , \qquad (6\text{-}4)$$

where e_s is the eccentricity of the satellite orbit. For an orbit having $e_s = 0.01$, this error amounts to 0.01 deg. This error is of a long term period of one year and therefore quite insignificant. For an inclined orbit the averaging scheme will eliminate the eccentricity error in the same manner. The residual error after averaging is the same order as (6-4) for orbits having inclination up to 10 deg. The position measurement scheme with averaging is therefore not sensitive to the variations of eccentricity and argument of the perigee.

The effect of the changes in the inclination and the ascending node can be computed from Eq. (5-7). The variation due to changes in these two elements is

$$\Delta\psi(t) = \frac{\partial\psi}{\partial i}\,\delta i + \frac{\partial\psi}{\partial\omega}\,\delta\omega \qquad (6\text{-}5)$$

Using $\psi(t)$ from Eq. (5-7) in above

$$\Delta\psi(t) \simeq \delta i \left(\frac{i}{2} + \frac{i^3}{6}\right) \sin \left(\frac{4\pi t}{365.27} + 2\omega\right) + 2\delta\omega \left(\frac{i^2}{4} + \frac{i^4}{24}\right) \cos \left(\frac{4\pi t}{365.27} + 2\omega\right) \quad . \tag{6-6}$$

For a synchronous satellite in the ecliptic plane, the major cause of deviation of the satellite orbit from the ecliptic plane is due to rotation of the ascending node of the orbit referred to the equator [4].

In Fig. 3, the normal to the satellite orbit, 0L, will execute a conical motion around the earth axis 0Z'. When the orbit normal is 0L, the orbit plane is coplanar with the ecliptic plane. The semi-vertical angle of the cone is η, which is the inclination between the ecliptic and the equatorial plane. Let the ascending node of the orbit referred to the equator and the 0Y direction be Ω. It follows from Fig. 3, that the inclination of the orbit referred to the ecliptic is

$$i = 2 \sin^{-1} \left(\sin \eta \sin \frac{\Omega}{2}\right) \quad . \tag{6-7}$$

Similarly, after some manipulations of the rotation matrix similar to the treatment in Section IV, it can be seen that the angle ω between 0X and the line of nodes of the orbit with the ecliptic plane is given by

$$\omega = \tan^{-1} \left(\frac{\sin \Omega}{1 + \cos \Omega} \cdot \frac{1}{\cos \eta}\right) \quad . \tag{6-8}$$

From Eq. (6-7) and (6-8)

$$\frac{di}{d\Omega} = \frac{\sin \eta \cos \frac{\Omega}{2}}{\sqrt{1 - \sin^2 \eta \sin^2 \frac{\Omega}{2}}} \quad , \tag{6-9}$$

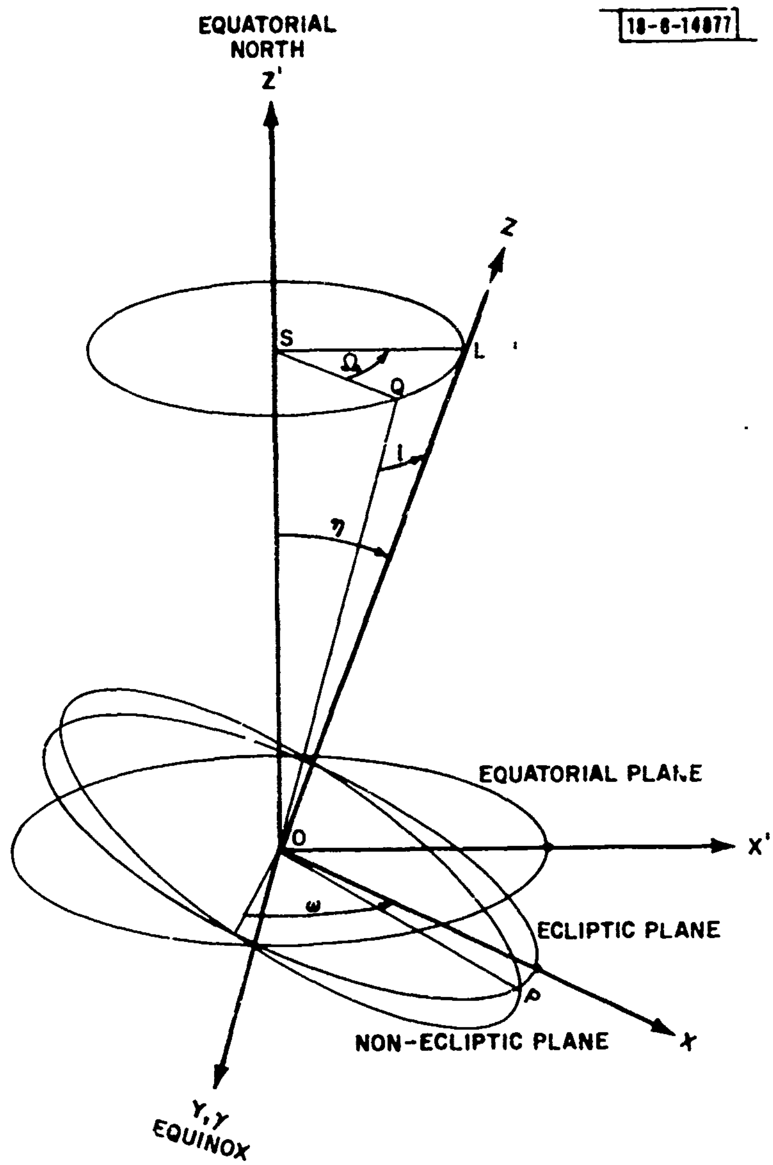

Fig. 3. Nutation of satellite orbit plane.

and

$$\frac{d\omega}{d\Omega} = \frac{(1+\cos\Omega)\cos\eta}{(1+\cos\Omega)^2\cos^2\eta + \sin^2\Omega} \quad . \tag{6-10}$$

From Ref. 4, the combined effect of the geopotential and lunar potential is to cause a precession of the line of nodes between the equatorial plane and the orbit plane at a rate of $d\Omega = -4.5$ deg/year. The desired values of the orbit elements referred to the equator at the beginning of the lifetime are

$$\begin{aligned} i &= 4.65 \text{ deg} \\ \Omega &= 11.6 \text{ deg} \ . \end{aligned} \tag{6-11}$$

With the above initial elements, the elements at the end of the five year lifetime will be

$$\begin{aligned} i &= 4.65 \text{ deg} \\ \Omega &= -11.6 \text{ deg} \ . \end{aligned} \tag{6-12}$$

If we make an allowance of 0.35 deg injection error, then the maximum inclination from the ecliptic at any time will be [7] 5.0 deg. Assuming $\Omega = 0$ deg and $\eta = 23.44°$ in Eq. (6-9) and (6-10), Eq. (6-6) gives

$$\Delta\psi(t) \simeq \delta\Omega \left\{ 0.034 \sin\left(\frac{4\pi t}{365.27} + 2\omega\right) + 0.008 \cos\left(\frac{4\pi t}{365.27} + 2\omega\right) \right\} \tag{6-13}$$

The error given by Eq. (6-13) is of a long term and can be allowed to build up and then corrected by resetting the synthesizer. It will be quite adequate to reset the synthesizer about three or four times over the five-year lifetime of the satellite.

VII. GENERATION OF THE SOLAR EPHEMERIS

It was seen in Section V that the maximum error due to discrete sampling of $\psi_1(\ell)$ at half day intervals is about 0.0084 deg and the maximum difference between two consecutive $\psi_1(\ell)$ is

$$\left[\delta\psi_1(\ell)\right]_{max} = 0.017 \text{ deg} \quad . \tag{7-1}$$

Also from (3-8) and (5-5)

$$\left[\psi_1(i)\right]_{max} \simeq 1.92 \text{ deg} \tag{7-2}$$

It is seen from above that it will require a much smaller memory to store the difference $\Delta\psi_1(\ell)$, than to store the function $\psi_1(\ell)$ itself. The function $\psi_1(\ell)$ can be generated from $\Delta\psi(\ell)$ by using

$$\psi_1(k) = \sum_{o}^{k} \Delta\psi_1(k) + \psi_1(o) \quad , \tag{7-3}$$

where $\Delta\psi(k) = \psi(k) - \psi(k-1)$, and $\psi(o)$ is the function $\psi_1(o)$ at the initial sampling instant. The above operation is performed by an adder. To generate the ephemeris for an inclined orbit, using Eq. (5-7), the function $\psi_1(k)$ has to be read out at proper phase and frequency and multiplied by the factor $\frac{1}{a}$. This required multiplication can be performed by controlling the rate of summation. For example, multipliers between zero to $\frac{1}{2}$ can be obtained by a jump summation

$$\frac{1}{u}\,\psi_1(k) = \sum_{j=o}^{k} \Delta\psi_1(j)\; \overline{\delta}_{\text{Int}\left(\frac{j}{u}\right),\ \text{Int}\left(\frac{j-1}{u}\right)} + \frac{\psi(o)}{u} \quad , \tag{7-4}$$

where Int(s) is the integral part of "s" and $\overline{\delta_{ij}} = \begin{matrix} 1 & i \neq j \\ 0 & i = j \end{matrix}$. Similarly for multipliers between $\frac{1}{2}$ to 1, a skip summation gives

$$\frac{u-1}{u}\,\psi_1(k) = \sum_{j=o}^{k} \Delta\psi_1(j)\,\delta_{\mathrm{Int}\left(\frac{j}{u}\right),\ \mathrm{Int}\left(\frac{j-1}{u}\right)} + \psi(o)\cdot\frac{u-1}{u} , \qquad (7\text{-}5)$$

where

$$\delta_{ij} = 1 - \bar{\delta}_{ij} .$$

Above relations are strictly valid only when the sampling interval is small as compared to the period of the function $\psi_1(k)$, and u is a small number. When u is a large integer the mu'.iplier is very small and the inaccuracies in using (7-4) and (7-5) for the inclination correction term are not significant For inclinations up to 14.8 deg only jump-summation is required. Since the number u can be only an integer, therefore, the inclination factor $I(i)$ can be selected only at discrete intervals. The maximum error due to discrete $I(i)$ is a function of the inclination i and it is given by

$$\psi_5(k) \approx \frac{1}{2}\cdot\frac{1}{u^2}\left[|a_1| + |a_2| + |a_3| + \ldots\right] . \qquad (7\text{-}6)$$

The multiplier required for various inclinations and the maximum error due to jump summation in the vicinity of this inclination is given in Table IV.

The error shown in Table IV has a long term period of one-half year. The short term error due to the above scheme of generating the inclination correction term depends directly on the accuracy of the storing of the differences $\Delta\psi_1(k)$.

VIII. DESIGN OF THE EPHEMERIS SYNTHESIZER

The least significant bit for the storage of the differences $\Delta\psi_1(k)$ is chosen to be 0.00275 deg. This makes the synthesizer compatible with the LES-8/9 earth sensor which has a least significant bit of 0.011 deg. With the above choice the error due to the discrete representation of angle will be less than 0.001375 deg. This contributes directly to the short term error of the synthesizer. The number representation is in binary two's complement. The memory necessary for storage of $\Delta\psi_1(k)$ is a 730 words of 4 bits, including the sign bit. A twelve bit adder is sufficient to add and store the ephemeris. The various design features of the ephemeris synthesizer are summarized in the Appendix A. Appendix B illustrates some ephemerides synthesized by a breadboard version of the synthesizer.

A block diagram of the synthesizer is shown in Fig. 4. A 768 word by 4 bit read-only memory is used for storing the function $\psi_1(k)$. The memory power is strobed. 1.5W instantaneous memory power is stored in a capacitor; therefore, the maximum power drained from the supply when reading the memory is only 0.3 mW. The synthesizer is provided with the capability of overriding the part of the logic which generates the $\frac{I}{a}\psi_1(2k + k_o)$ term and generating only the $\psi_1(k)$ term.

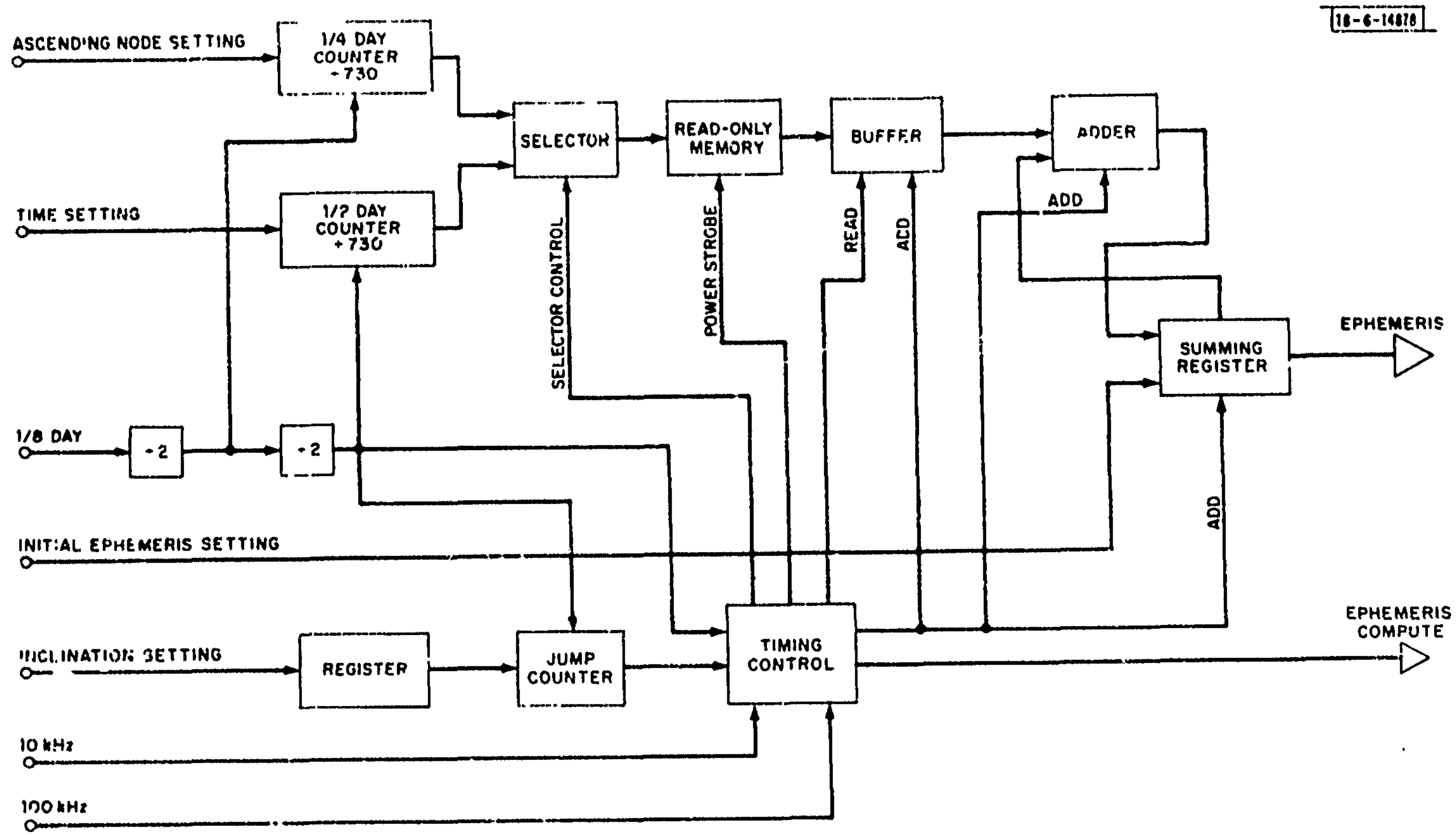

Fig. 4. Block diagram of the ephemeris synthesizer.

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APPENDIX A

DESIGN SPECIFICATIONS OF THE SOLAR EPHEMERIS SYNTHESIZER

Satellite orbit	Circular synchronous* 0 to 10 deg inclination with ecliptic
Satellite attitude	The ephemerides are valid for a satellite having no attitude errors*.
Short term error	< 0.001 deg
Long term error	0.050 deg
Resettings (command) required	Four (1) Time of resetting 12 bit (2) Ascending node 12 bit (3) Inclination 8 bit (4) Initial ephemeris 12 bit
Outputs	(1) 12 bit solar ephemeris in two's complement (2) Ephemeris compute signal -- indicates time during which the ephemeris is computed Other outputs available for system check and testing
Inputs	(1) 100 kHz clock (2) 10 kHz clock (3) 1/8 day clock
Max. power required	700 mW at + 5V ± 10%
Size	0.6" × 4-3/4" × 6-1/4"
Weight	0.72 lbs.

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* Additional corrections required for eccentricity and attitude errors are done by using the output of two sun transit sensors and the infrared earth sensors. The computation of these corrections are done in the longitude logic.

APPENDIX B

EXAMPLES OF SOLAR EPHEMERIS SYNTHESIZED FOR VARIOUS ORBITS

In the next few pages, some examples of the solar ephemeris synthesized for various orbits are shown. The synthesizer is a mechanization of the Eq. (5-7).

$$\psi(t) = \psi_1 \left(\frac{2\pi t}{365.27}\right) + \frac{I}{a_1}\, \psi_1\left(\frac{4\pi t}{365.27} + 2\omega\right) \quad \ldots \qquad \text{(B-1)}$$

where,

$$I = \left(\frac{i^2}{4} + \frac{i^4}{24}\right)$$

i = inclination of the orbit with the ecliptic plane

ω = angle measured along the ecliptic between the direction of perigee and the ascending node of the orbit.

t = time measured from the instant of the perigee passage.

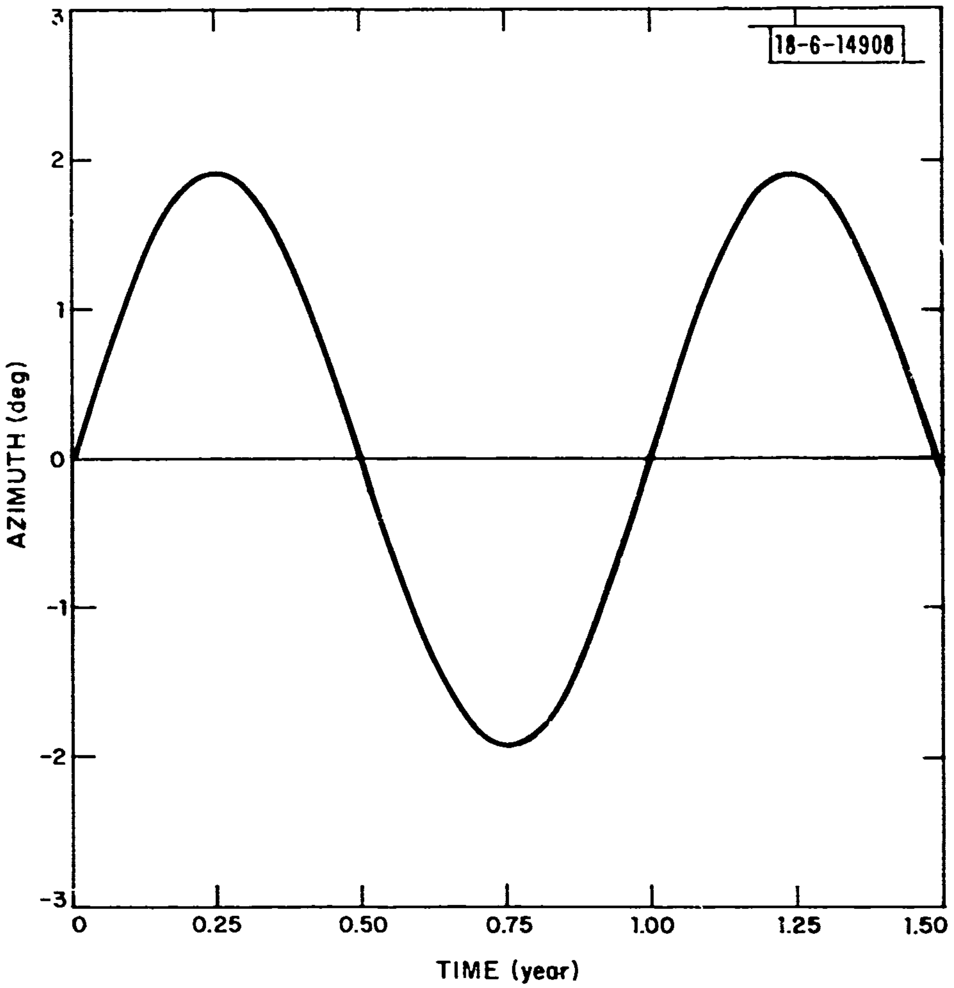

Fig. B-1. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 0.00^{o}$, $\omega = 0^{o}$, $e = o$, $a = R_{syn}$.

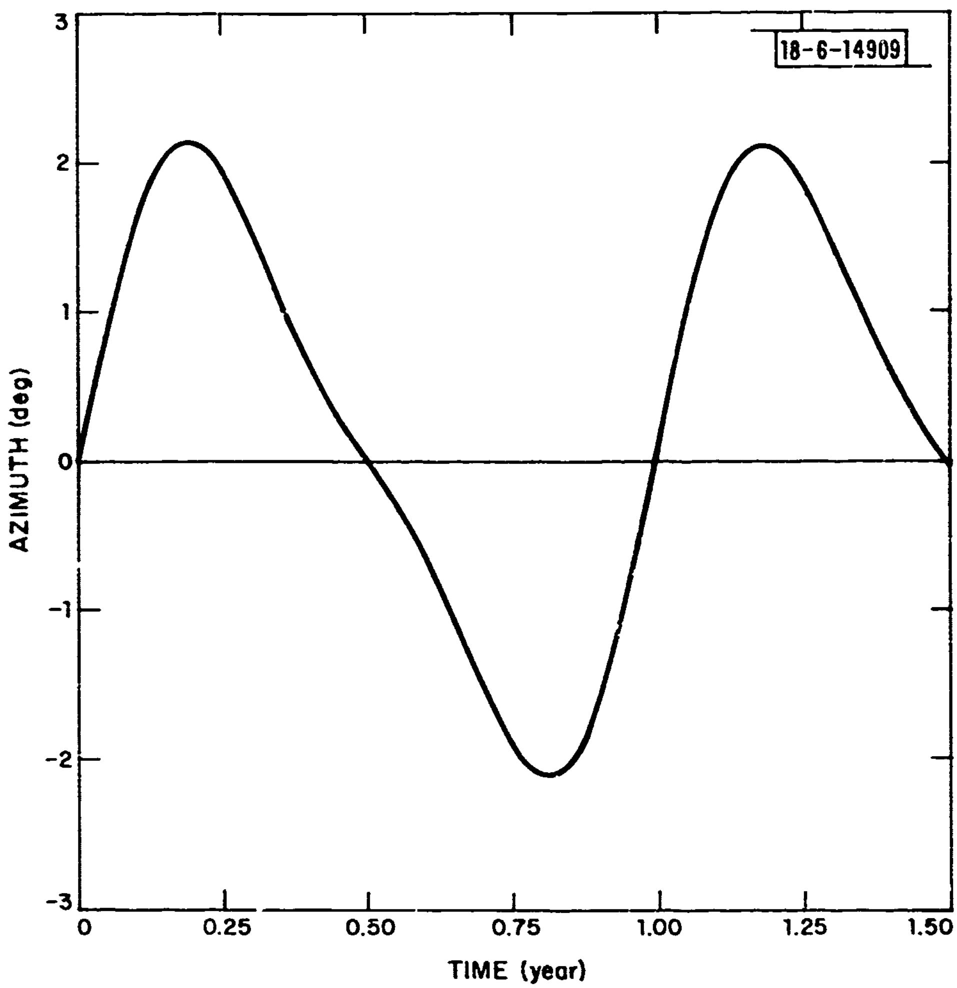

Fig. B-2. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 10.50^{o}$, $\omega = o^{o}$, $e = o$, $a = R_{syn}$.

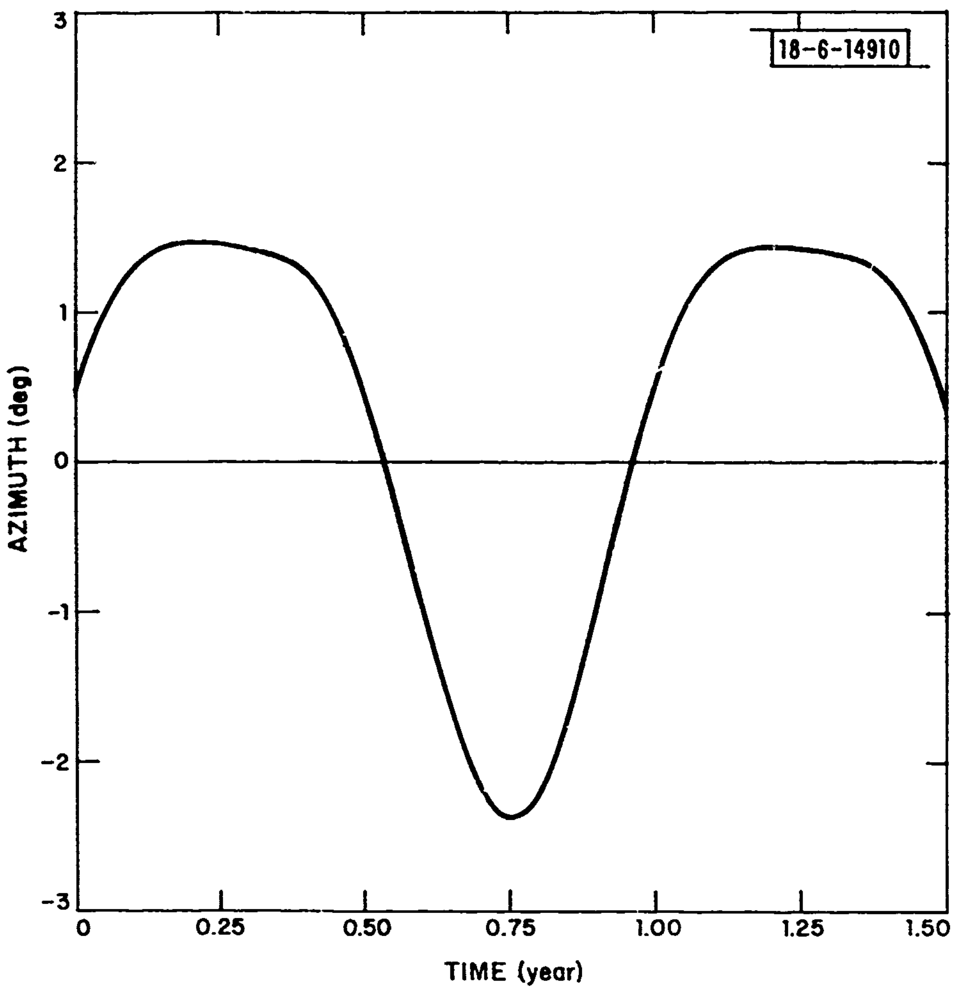

Fig. B-3. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 10.50^{\circ}$, $\omega = 45^{\circ}$, $e = o$, $a = R_{syn}$.

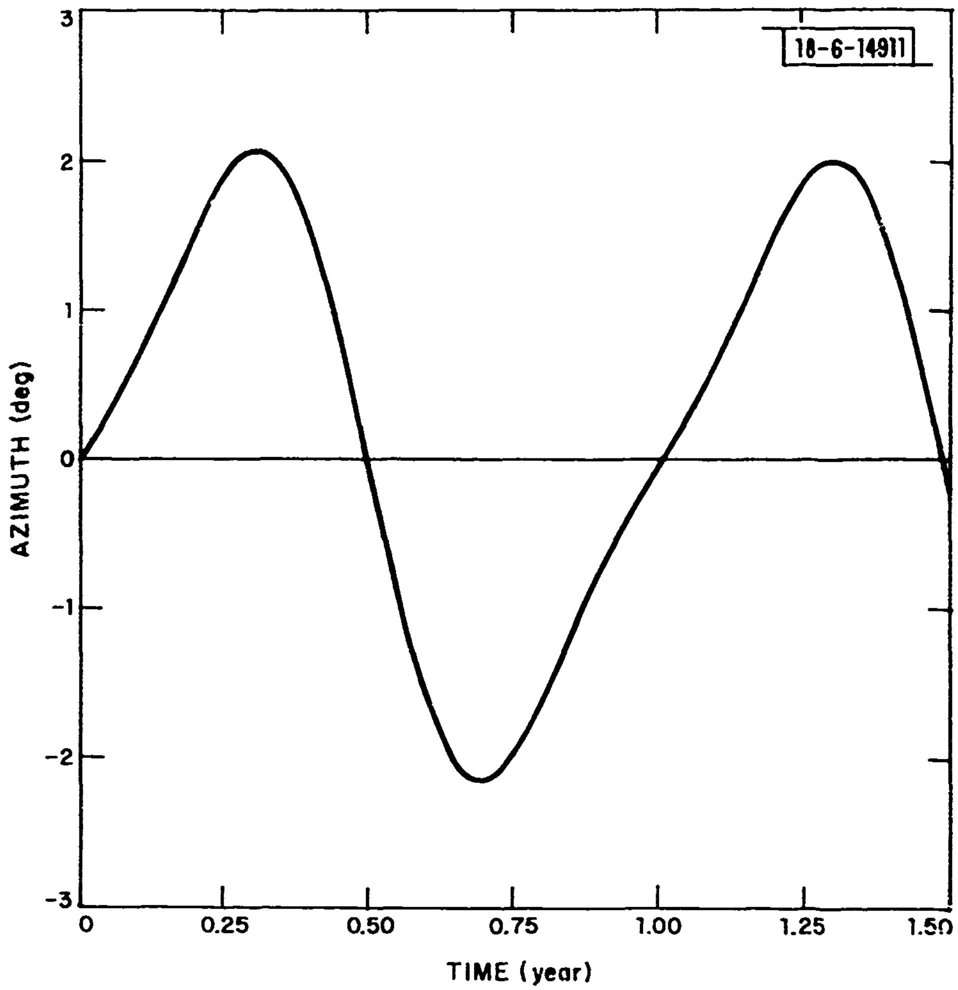

Fig. B-4. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 10.50^o$, $\omega = 90^o$, $e = o$, $a = R_{syn}$.

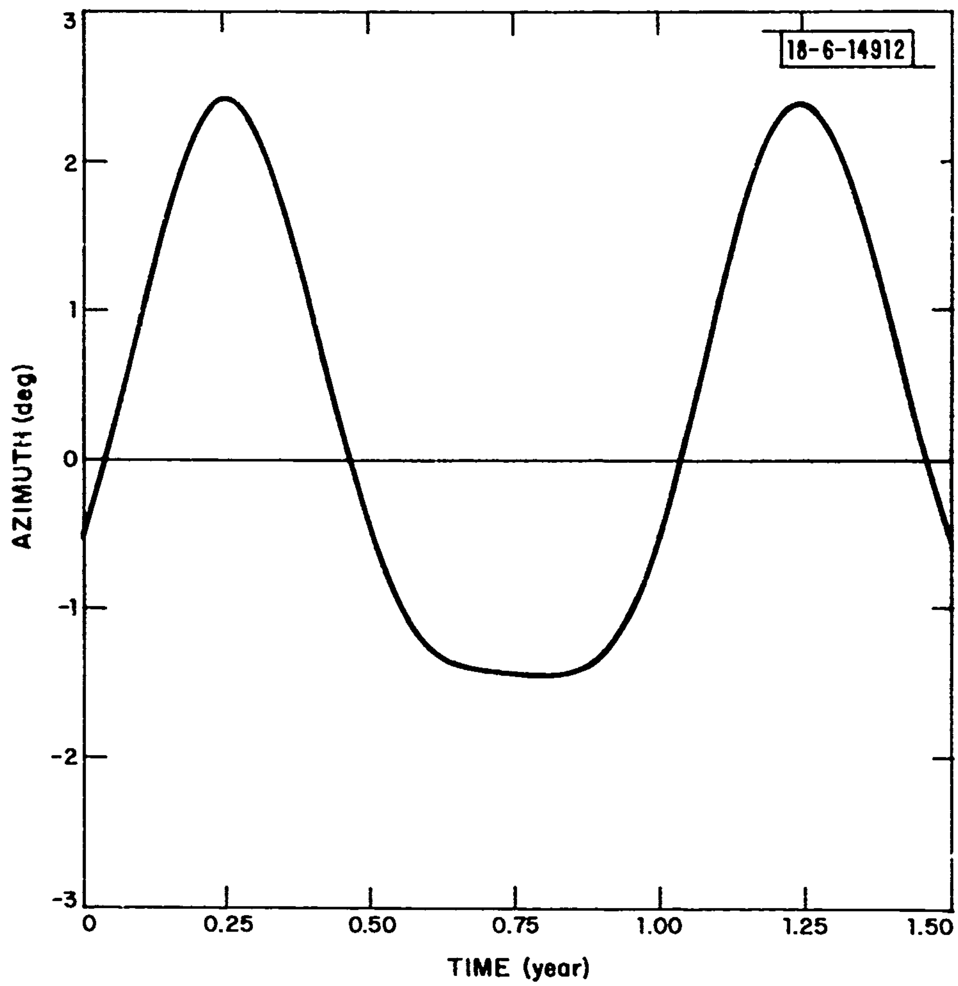

Fig. B-5. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 10.50^{\circ}$, $\omega = 135^{\circ}$, $e = o$, $a = R_{syn}$.

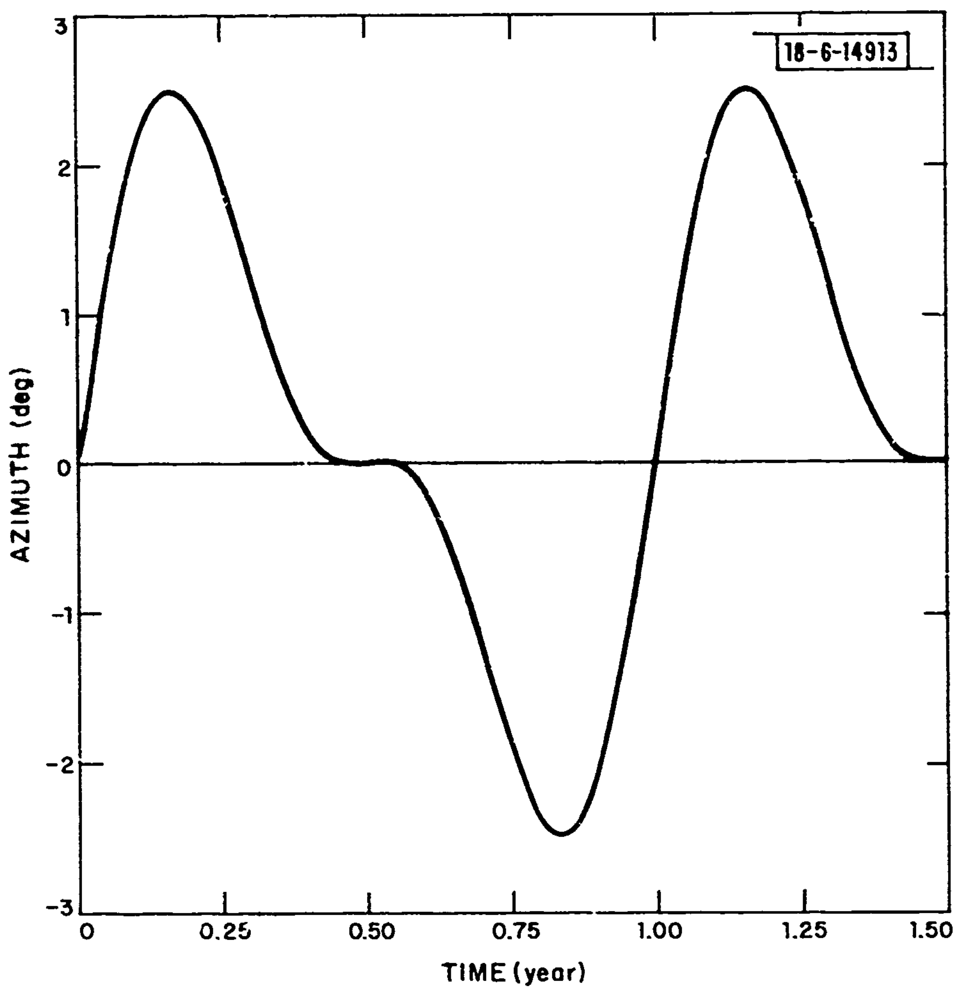

Fig. B-6. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 14.85^{\circ}$, $\omega = 0^{\circ}$, $e = o$, $a = R_{syn}$.

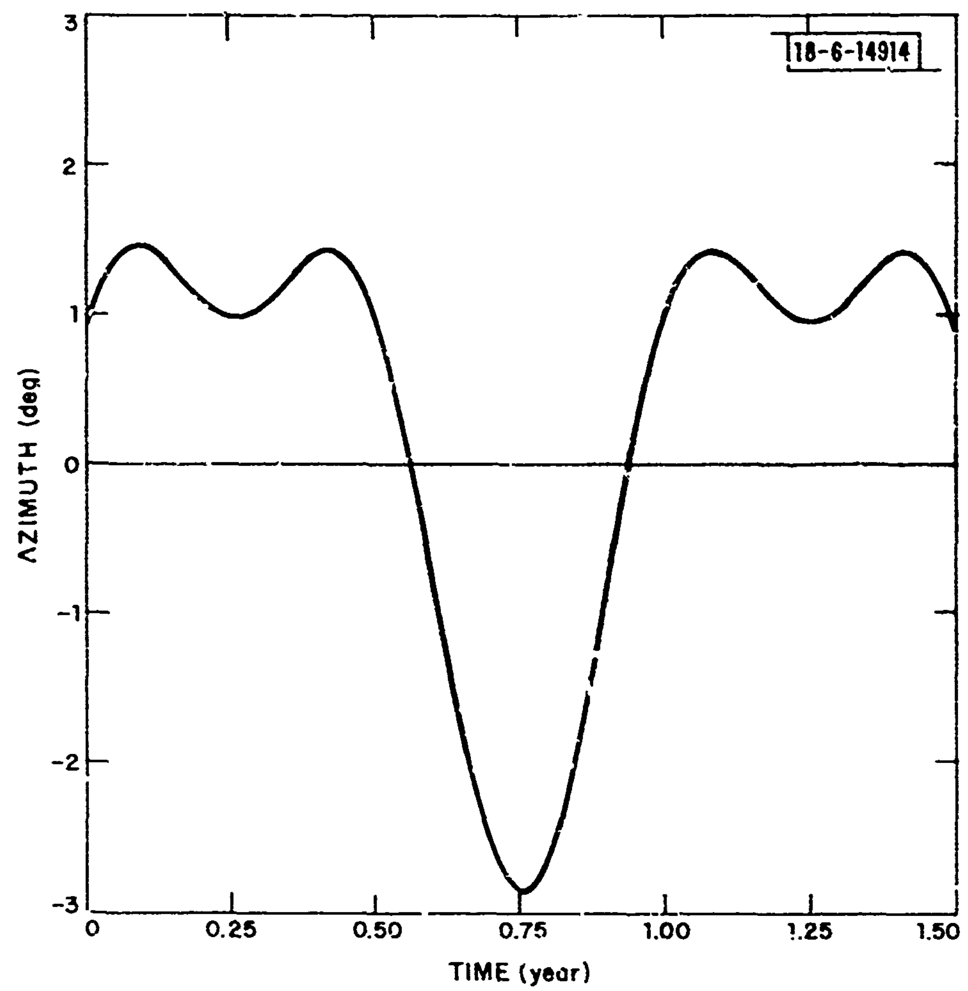

Fig. B-7. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 14.85^{\circ}$, $\omega = 45^{\circ}$, $e = o$, $a = R_{syn}$.

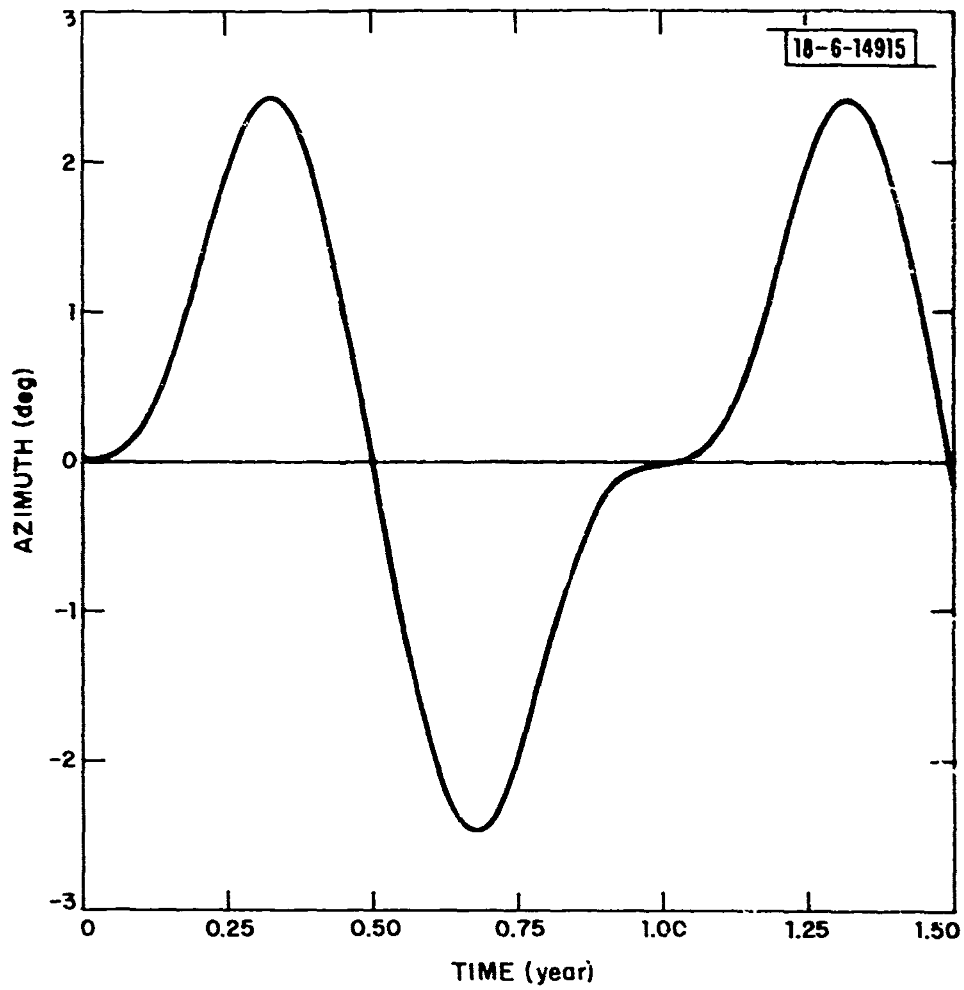

Fig. B-8. Solar ephemeris for a synchronous satellite. No attituae errors, $i = 14.85^{o}$, $\omega = 90^{o}$, $e = o$, $a = R_{syn}$.

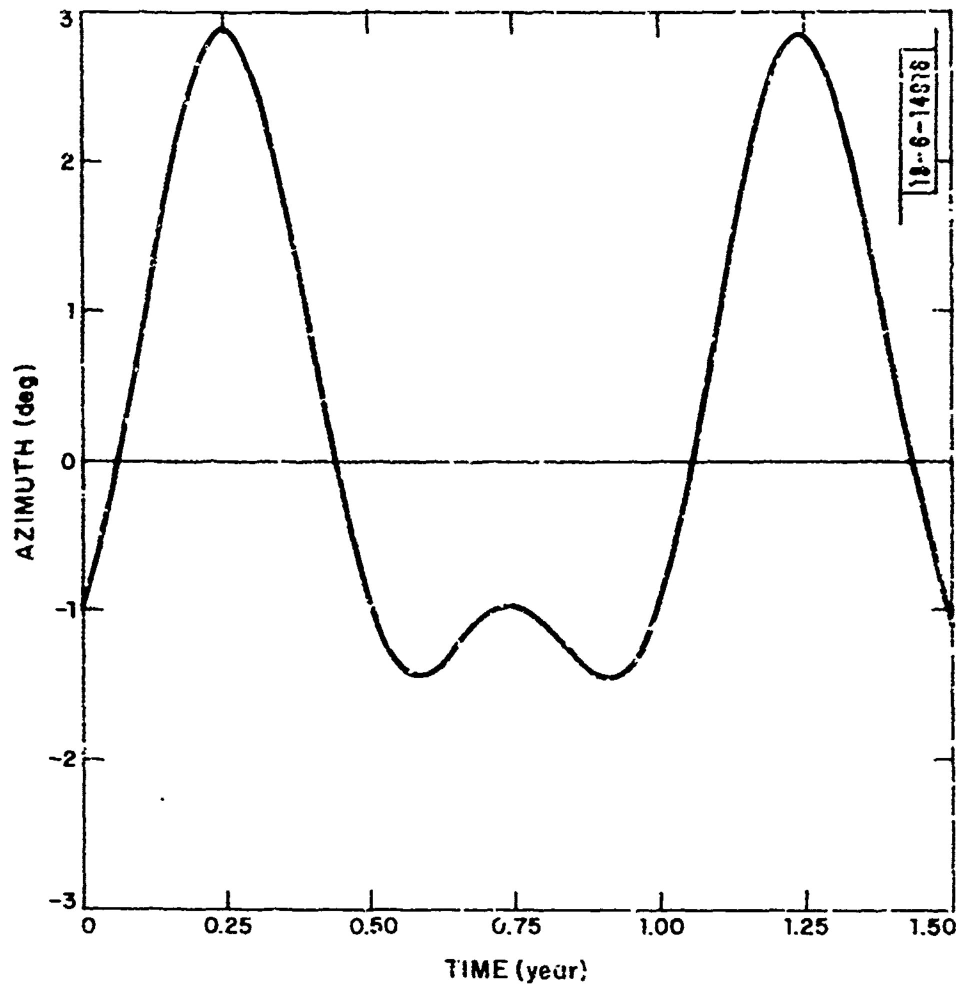

Fig. B-9. Solar ephemeris for a synchronous satellite. No attitude errors, $i = 14.85^{\circ}$, $\omega = 135^{\circ}$, $e = o$, $a = R_{syn}$.